

# Set Trimming Procedure for the Design Optimization of Shell and Tube Heat Exchangers

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**ABSTRACT:** In almost all previous approaches for the design optimization of shell-and-tube heat exchangers, global optimality was not guaranteed. Recognizing that the geometric components are discrete due to its physical nature (e.g., number of baffles) or commercial standards (e.g., tube diameter), purely integer linear models or mixed integer linear models using the Kern or the Bell-Delaware models were developed in previous work and solved globally using mathematical programming. The time used to solve these models was relatively small but still too large when repeated calculations are needed. We show in this article that the use of Set Trimming avoids using mathematical programming (MINLP or MILP) and reduces the computational time by at least 2 orders of magnitude.



## 1. INTRODUCTION

Shell and tube heat exchangers are the most popular thermal equipment in chemical process industries because they are reliable, versatile, and robust.<sup>1</sup> Therefore, the optimization of the design of shell and tube heat exchangers can bring relevant reductions of the capital costs associated with grassroots designs and plant retrofits. This importance has motivated a large number of developments of new design procedures for shell and tube heat exchangers. These prior works can be organized into three main classes: heuristic and enumeration procedures, stochastic methods, and mathematical programming.

Heuristic and enumeration procedures involve algorithms that explore the search space through the systematic analysis of solution candidates. Some of these approaches are based on a graphical representation of the search space<sup>2</sup> and others involve different kinds of enumeration procedures, where a systematic sequence of solution candidates is generated until the best alternative is found.<sup>3</sup>

Stochastic methods encompass a set of different algorithmic alternatives, for example, simulated annealing,<sup>4</sup> genetic algorithms,<sup>5</sup> particle swarm optimization,<sup>6</sup> ant colony optimization,<sup>7</sup> cuckoo search,<sup>8</sup> firefly algorithm,<sup>9</sup> and so forth. Despite the capacity of the stochastic optimization methods to avoid local optima during the search, global optimality cannot be guaranteed. In addition, it is well-known that the success of this type of algorithms is strongly tied to high human intervention to tune their parameters.

Mathematical programming, in turn, is represented by a set of algorithms based on rigorous optimality conditions of a mathematical model. Because of the nonconvex formulation of the design problem in its original form, many of the

alternatives developed for shell and tube exchangers such as NLP<sup>10</sup> and MINLP<sup>11</sup> can end trapped in local optima if the solver used is local. More recently, our research group has developed techniques which provided linear representations of the design problem, therefore allowing the rigorous identification of the global optimum. These alternatives employ integer linear or mixed integer linear models (ILM and MILMs) that are the result of rigorous nonlinear model reformulations rooted on the fact that the geometric components (e.g., tubes, baffles) are represented by discrete choices. The corresponding formulations correspond to MILP (using Kern<sup>12</sup> and Bell<sup>13</sup> models) and ILP problems.<sup>14</sup>

The computational times associated with the linear formulations mentioned above were acceptable for individual design problems, but they limit the utilization of the proposed approaches when a large set of design problems must be solved simultaneously (e.g., heat exchanger network synthesis including equipment design). Aiming at reducing the time needed to reach the global optimum of the shell and tube heat exchanger design problem, this paper explores the utilization of a new search algorithm called Set Trimming and compares its performance with the aforementioned linear models. The technique was first proposed by Gut and Pinto<sup>15</sup> in a specialized context and later formalized by Costa and

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Bagajewicz.<sup>16</sup> Set trimming employs the set of inequality constraints to obtain the optimum through the sequential generation of smaller subsets where the solution can be found by simple enumeration, sorting, or solving much smaller problems.

It is important to distinguish Set Trimming from Domain Reduction and Constraint Propagation, the latter being considered by some as a special case of the former.<sup>17</sup> Despite that Domain Reduction and Set Trimming explore the problem constraints to reduce the search space, the current utilization of Set Trimming is not equivalent to conventional Domain Reduction procedures. Domain Reduction techniques reduce the interval that delimits each problem variable range (i.e., the box that contains the feasible region) and is a preprocessing technique for an optimization procedure that inevitably follows, typically mathematical programming. Instead, Set Trimming does not operate reducing a box and exploits the discrete nature of the variables. It explores the search space using a combinatorial representation of the domain, not using the original variables, but a set of discrete options (each set of discrete values is a solution candidate), that is, combinations of instances of the original discrete independent variables. Another differentiating aspect is that Set Trimming does not necessarily demand to be followed by a complex optimization. In particular, in the proposed design problem of this article the identification of the optimal solution only involves a simple inspection, avoiding the use of any additional mathematical programming procedure.

This article is organized as follows. We first present two shell and tube heat exchanger models of increasing complexity: the Kern model followed by the Bell-Delaware model. Next, we present the formulation of the design optimization problem and the set trimming procedure. We finish the paper with examples of application of the proposed technique to the solution of the optimal design of shell and tube heat exchangers, showing that the use of set trimming can solve these problems in only a fraction of the time needed by mixed-integer programming techniques.

## 2. SHELL AND TUBE HEAT EXCHANGER MODELS

The design optimization of shell and tube heat exchangers referred to in this paper is applied to streams without phase change associated with a single E-shell type. However, the proposed solution technique can also be applied to other thermal services.

Two heat exchanger models are explored, identified here according to the shell-side flow model adopted: Kern<sup>18</sup> and Bell-Delaware.<sup>19</sup> These options are the alternatives explored in the previous papers in the literature. More modern alternatives are based on proprietary data (stream analysis method) or involve a prohibitive computational effort (CFD).<sup>20</sup> The Kern model is associated with the Dittus-Boelter correlation for the evaluation of the heat transfer coefficient<sup>21</sup> and the Darcy-Weisbach equation for evaluation of the pressure drop<sup>22</sup> in the tube-side flow. The Bell-Delaware model is associated with a set of models for evaluation of the heat transfer coefficient in the tube-side flow for different flow regimes, encompassing the theoretical solution for fully developed laminar flow and the empirical correlations of Gnielinski, Hausen, Sieder, and Tate.<sup>21</sup> The pressure drop for the tube-side flow is evaluated using the Darcy-Weisbach equation, also including a disjunction to include the evaluation of the friction factor for all flow regimes.<sup>22</sup>

We present below the main equations that are common to both models investigated. The model parameters are identified with a symbol  $\wedge$  on top. The complete and detailed description of each model is available in the [Supporting Information](#).

**2.1. Geometric Constraints.** The ratio between the tube length ( $L$ ) and shell diameter ( $D_s$ ) must obey the following bounds<sup>23</sup>

$$L \geq 3 D_s \quad (1)$$

$$L \leq 15 D_s \quad (2)$$

Additionally, the baffle spacing ( $l_{bc}$ ) must be bounded in relation to the shell diameter ( $D_s$ )<sup>24</sup>

$$l_{bc} \geq 0.2 D_s \quad (3)$$

$$l_{bc} \leq 1.0 D_s \quad (4)$$

**2.2. Velocity Bounds.** Lower bounds on flow velocities are established to avoid fouling and upper bounds are established to avoid vibration and erosion

$$v_s \geq v_{smin} \quad (5)$$

$$v_s \leq v_{smax} \quad (6)$$

$$v_t \geq v_{tmin} \quad (7)$$

$$v_t \leq v_{tmax} \quad (8)$$

**2.3. Reynolds Numbers Bounds.** Bounds on the Reynolds number may be applied according to the validity of the thermofluid dynamic correlations

$$Re_t \geq Re_{tmin} \quad (9)$$

$$Re_t \leq Re_{tmax} \quad (10)$$

$$Re_s \geq Re_{smin} \quad (11)$$

$$Re_s \leq Re_{smax} \quad (12)$$

**2.4. Pressure Drop Bounds.** The pressure drops in the shell-side and tube-side ( $\Delta P_s$  and  $\Delta P_t$ ) must be bounded according to previously maximum established values ( $\Delta P_{sdisp}$  and  $\Delta P_{tdisp}$ ), many times indirectly related to a trade-off between capital and operating costs, and/or operational restrictions

$$\Delta P_s \leq \Delta P_{sdisp} \quad (13)$$

$$\Delta P_t \leq \Delta P_{tdisp} \quad (14)$$

**2.5. Heat Transfer Rate Equation.** The relation between the heat load ( $\dot{Q}$ ) and the required heat transfer area ( $A_{req}$ ) is based on the LMTD method

$$\dot{Q} = U A_{req} \Delta T_{lm} F \quad (15)$$

where  $U$  is the overall heat transfer coefficient,  $\Delta T_{lm}$  is the logarithmic mean temperature difference for the counter-current configuration, and  $F$  is the correction factor of the mean temperature difference.

The expression for evaluation of the overall heat transfer coefficient is

$$U = \frac{1}{\frac{dte}{dti} + \frac{\widehat{R}ft}{dti} + \frac{dte \ln\left(\frac{dte}{dti}\right)}{2 k_{tube}} + \widehat{R}fs + \frac{1}{hs}} \quad (16)$$

where  $d_{ti}$  and  $d_{te}$  are the inner and outer tube diameters;  $h_t$ ,  $\widehat{R}_{ft}$ ,  $h_s$ ,  $\widehat{R}_{fs}$  are the heat transfer coefficients and fouling factors for the tube-side and shell-side, respectively; and  $k_{tube}$  is the thermal conductivity of the tube wall.

The logarithmic mean temperature difference for the countercurrent configuration is given by

$$\widehat{\Delta T_{lm}} = \frac{(\widehat{T}_{hi} - \widehat{T}_{co}) - (\widehat{T}_{ho} - \widehat{T}_{ci})}{\ln\left(\frac{\widehat{T}_{hi} - \widehat{T}_{co}}{\widehat{T}_{ho} - \widehat{T}_{ci}}\right)} \quad (17)$$

where  $\widehat{T}_{hi}$ ,  $\widehat{T}_{ho}$ ,  $\widehat{T}_{ci}$ ,  $\widehat{T}_{co}$  are the hot and cold streams inlet and outlet temperatures, respectively.

The correction factor of the mean temperature difference is equal to 1 for one tube-side pass and is equal to the following expression for an even number of tube passes

$$F = \frac{(\hat{R}^2 + 1)^{0.5} \ln\left(\frac{(1 - \hat{P})}{(1 - \hat{R}\hat{P})}\right)}{(\hat{R} - 1) \ln\left(\frac{2 - \hat{P}(\hat{R} + 1 - (\hat{R}^2 + 1)^{0.5})}{2 - \hat{P}(\hat{R} + 1 + (\hat{R}^2 + 1)^{0.5})}\right)} \quad (18)$$

where  $\hat{R}$  and  $\hat{P}$  depend on the end temperatures

$$\hat{R} = \frac{\widehat{T}_{hi} - \widehat{T}_{ho}}{\widehat{T}_{co} - \widehat{T}_{ci}} \quad (19)$$

$$\hat{P} = \frac{\widehat{T}_{co} - \widehat{T}_{ci}}{\widehat{T}_{hi} - \widehat{T}_{ci}} \quad (20)$$

The heat transfer area of the exchanger ( $A$ ) corresponds to the surface area of the set of tubes and must be higher than the required heat transfer area according to a given design margin, expressed through an "excess area" ( $\widehat{A}_{exc}$ ) (this parameter aims to compensate the uncertainties associated with the thermohydraulic correlations, physical properties, fouling factors, etc.)

$$A = N_{tt} \pi d_{te} L \quad (21)$$

$$A \geq \left(1 + \frac{\widehat{A}_{exc}}{100}\right) A_{req} \quad (22)$$

where  $N_{tt}$  is the total number of tubes of the heat exchanger.

### 3. OPTIMIZATION USING SET TRIMMING

**3.1. Problem Formulation.** The design problem corresponds to the minimization of the heat transfer surface according to available maximum pressure drops.

The basic geometric variables employed to characterize a shell-and-tube heat exchanger in the optimization are inner and outer tube diameters ( $d_{ti}$  and  $d_{te}$ ), shell diameter ( $D_s$ ), tube length ( $L$ ), number of baffles ( $N_b$ ), number of tube passes ( $N_{tp}$ ), tube pitch ratio ( $rp$ ), and tube layout ( $lay$ ). In the Bell-Delaware model, the baffle cut ratio ( $Bc$ ) is also included in the set of design variables (the Kern model is limited to heat exchangers with a baffle cut ratio of 25%).

These variables determine a discrete search space, because of their physical nature, for example, number of baffles, number of tube passes, and tube layout, or because of manufacturing standards, for example, tube diameter, shell diameter, tube length, and tube pitch ratio. The baffle cut ratio is in principle and at first glance, a continuous variable, but final fabrication dimensions require discrete values. Therefore, a solution

candidate (i.e., a heat exchanger design option) corresponds to a set of discrete values of the design variables and the search space is composed of all possible combinations of these discrete values. Set Trimming takes advantage of this.

Another formulation of the heat exchanger design problem corresponds to the minimization of the total annualized cost (TAC), which encompasses capital and operating costs. The problem formulation is similar to the minimization of the area, but the constraints related to the pressure drops (eqs 13 and 14) can be omitted, because this alternative explores the trade-off between the cost associated with the energy consumption for the streams' flow (operational costs) and the cost associated with the heat transfer area (capital costs). If left, they serve some purpose other than economics (like process-related constraints).

**3.2. Set Trimming Procedure.** In a recent article,<sup>16</sup> the technique of set trimming was proposed as a means to perform a screening of the space of candidate solutions for the basic design of process equipment and the identification of the global optimum. Here, we refer to a candidate solution as a set of choices of the geometric discrete variables in such a way that their knowledge is sufficient to determine the exchanger performance, that is, in this case if its area is larger than  $A_{req}$  as per eq 22 and if the pressure drops are limited by the available values. Although Costa and Bagajewicz<sup>16</sup> formalized it, the technique can be traced back to an ad-hoc solution procedure for plate heat exchangers developed by Gut and Pinto.<sup>15</sup>

The optimization procedure consists of progressively checking sets of possible candidates employing inequality constraints and eliminating the ones that are not feasible. By the end, after testing all inequality constraints the remaining exchangers are feasible and the final result is the one which has the smaller area (or total annualized cost) within this final set. Since only infeasible options are eliminated during the trimming and the final selection picks the best alternative among all feasible candidates, global optimality is assured.

The analysis of a given inequality constraint may involve the evaluation of some equality constraints from the heat exchanger model. For example, for a given set of values of the geometric design variables the equality constraints of the heat exchanger thermofluid dynamic model allow the evaluation of all continuous variables for the tube-side and shell-side streams: flow velocities, Reynolds numbers, Nusselt numbers, heat transfer coefficients, overall heat transfer coefficient, friction factors, and pressure drops.

Instead of testing all solution candidates using all inequality constraints, a better option involves the application of the constraints sequentially, where only the feasible solution candidates from a constraint are tested in the next one. The order in which the constraints are tested takes into account the complexity of the constraint in terms of how many computations it involves.

The steps associated with each trimming for the design of shell and tube heat exchangers are displayed below, where, according to the sequential pattern, each set is defined as a subset of the previous one. The order of the sequence of trimmings was established aiming at locating the more complex constraints (which would consume more time due to the higher number of mathematical operations) at the end of the search, where there is a smaller number of solution candidates. The initial search space corresponds to the set  $S$ . The procedure is represented graphically in Figure 1.

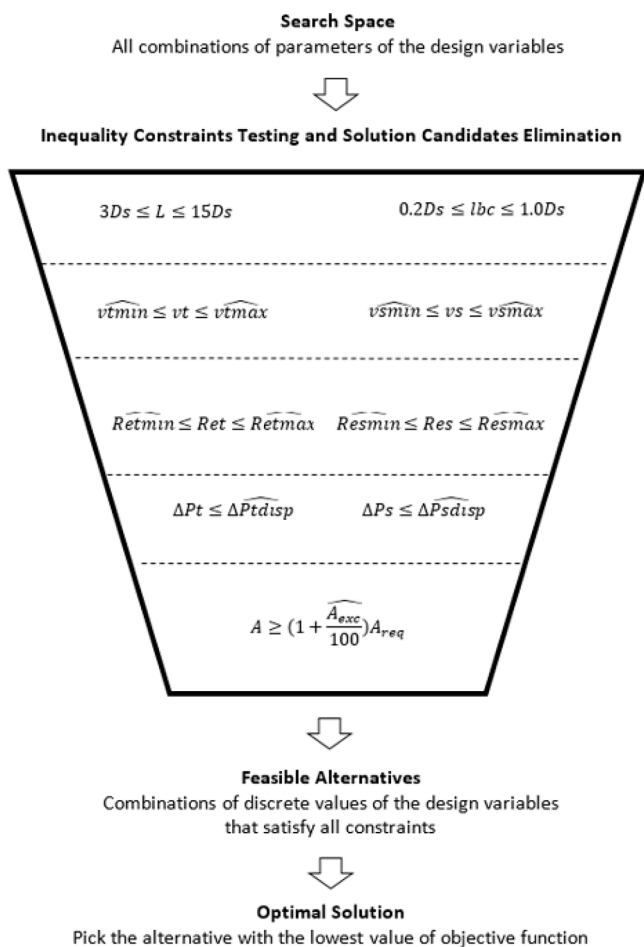


Figure 1. Set trimming procedure.

Step 1 (geometric trimming): We determine which heat exchangers are feasible for the minimum and maximum tube length/shell diameter ratio and the minimum and maximum baffle spacing

$$S_{LDmin} = \{srow \in S/L \geq 3 \text{ } Ds \text{ is satisfied}\} \quad (23)$$

$$S_{LDmax} = \{srow \in S_{LDmin}/L \leq 15 \text{ } Ds \text{ is satisfied}\} \quad (24)$$

$$S_{LNbmin} = \{srow \in S_{LDmax}/lbc \geq 0.2 \text{ } Ds \text{ is satisfied}\} \quad (25)$$

$$S_{LNbmax} = \{srow \in S_{LNbmin}/lbc \leq 1.0 \text{ } Ds \text{ is satisfied}\} \quad (26)$$

Step 2 (flow velocity trimming): We determine which heat exchangers are feasible for the minimum and maximum tube-side and shell-side velocities

$$S_{vtmin} = \{srow \in S_{LNbmax}/vt \geq \widehat{vtmin} \text{ is satisfied}\} \quad (27)$$

$$S_{vtmax} = \{srow \in S_{vtmin}/vt \leq \widehat{vtmax} \text{ is satisfied}\} \quad (28)$$

$$S_{vsmin} = \{srow \in S_{vtmax}/vs \geq \widehat{vsmin} \text{ is satisfied}\} \quad (29)$$

$$S_{vsmax} = \{srow \in S_{vsmin}/vs \leq \widehat{vsmax} \text{ is satisfied}\} \quad (30)$$

Step 3 (Reynolds number trimming): We determine which heat exchangers are feasible for the tube-side and shell-side Reynolds number constraints

$$S_{Retmin} = \{srow \in S_{vsmax}/Ret \geq \widehat{Retmin} \text{ is satisfied}\} \quad (31)$$

$$S_{Retmax} = \{srow \in S_{Retmin}/Ret \leq \widehat{Retmax} \text{ is satisfied}\} \quad (32)$$

$$S_{Resmin} = \{srow \in S_{Retmax}/Res \geq \widehat{Resmin} \text{ is satisfied}\} \quad (33)$$

$$S_{Resmax} = \{srow \in S_{Resmin}/Res \leq \widehat{Resmax} \text{ is satisfied}\} \quad (34)$$

Step 4 (pressure drop trimming): We determine which heat exchangers are feasible for the maximum shell-side and tube-side pressure drops

$$S_{DPTmax} = \{srow \in S_{Resmax}/\Delta Pt \leq \widehat{\Delta Ptdisp} \text{ is satisfied}\} \quad (35)$$

$$S_{DPsmax} = \{srow \in S_{DPTmax}/\Delta Ps \leq \widehat{\Delta Psdisp} \text{ is satisfied}\} \quad (36)$$

Step 5 (Required area trimming): We determine which heat exchangers are feasible for the minimum required area

$$S_{Amin} = \left\{srow \in S_{DPsmax}/A \geq \left(1 + \frac{\widehat{Aexc}}{100}\right)Areq \text{ is satisfied}\right\} \quad (37)$$

Step 6 (Determining the optimal heat exchanger): The exchanger with the smaller area/total annualized cost within the set  $S_{Amin}$  (i.e., the feasible region of the design problem) is the optimal solution.

It important to observe that this procedure does not need an initial estimate, but if a feasible solution is available, then the search could be accelerated through the insertion of an additional trimming that would eliminate candidates with a larger objective function value before starting. Such an initial solution must be generated very quickly in our case.

#### 4. RESULTS

The performance of the set trimming approach for the design of shell and tube heat exchangers was tested using a set of 10 design problems portrayed by Gonçalves et al.<sup>12</sup> (a complete description of each problem is available in the [Supporting Information](#)). The search space of the optimization is

Table 1. Discrete Values of the Design Variables

variable	values
outer tube diameter, $dte$ (m)	0.01905, 0.02540, 0.03175, 0.03810, 0.05080
tube length, $L$ (m)	1.2195, 1.8293, 2.4390, 3.0488, 3.6585, 4.8768, 6.0976
number of baffles, $Nb$	1, 2, ..., 20
number of tube passes, $Ntp$	1, 2, 4, 6
tube pitch ratio, $rp$	1.25, 1.33, 1.50
shell diameter, $Ds$ (m)	0.7874, 0.8382, 0.8890, 0.9398, 0.9906, 1.0668, 1.143, 1.2192, 1.3716, 1.524
tube layout, $lay$	1 (30° layout), 2 (90° layout)
baffle cut ratio, $Bc$	0.25



Table 2. Set Trimming versus Mathematical Programming: Area Minimization

example	Kern model			Bell-Delaware model		
	area (m <sup>2</sup> )	time (s) ILP	time (s) trimming	area (m <sup>2</sup> )	time (s) MILP	time (s) trimming
1	624.59	11.14	0.33	523.90	312.16	1.64
2	319.67	10.75	0.33	274.06	216.36	1.47
3	199.34	10.84	0.26	158.19	129.60	1.24
4	872.9	11.65	0.27	711.60	106.94	1.14
5	143.78	10.89	0.26	122.61	380.88	1.11
6	332.14	10.79	0.33	283.55	294.05	1.63
7	207.64	11.44	0.28	171.00	152.78	1.31
8	915.23	11.27	0.33	840.86	282.62	1.60
9	287.48	11.10	0.34	219.19	356.79	1.63
10	327.76	11.17	0.31	237.93	113.90	1.16

Table 3. Number of Solution Candidates Remaining after each Set Trimming Step (Bell-Delaware Model)

example	start	geometric	tube-side velocity	shell-side velocity	tube-side Reynolds number	shell-side Reynolds number	tube-side pressure drop	shell-side pressure drop	required area
1	168000	61560	24441	16888	16888	16888	15829	15001	426
2	168000	61560	25287	16395	16395	16395	11041	10810	724
3	168000	61560	9128	6499	6499	6434	6384	6376	913
4	168000	61560	25392	3679	3679	3346	2973	1662	19
5	168000	61560	13662	6363	6363	3746	3476	2892	1562
6	168000	61560	22968	15738	15738	15584	14872	14660	1587
7	168000	61560	13662	9638	9638	9638	9192	6929	1021
8	168000	61560	20726	14891	14891	14891	14736	14447	80
9	168000	61560	24275	16706	16706	16160	15886	15195	4663
10	168000	61560	25268	10661	10661	2444	2444	2431	1617

characterized by the set of standard values of the design variables displayed in Table 1, typical values of the industrial practice.<sup>25</sup> The tube thickness employed in all examples was 0.001225 m.

The computational times employed to solve these problems using set trimming are compared with mathematical programming based on MILP<sup>13</sup> and ILP.<sup>14</sup> All computational tests were carried out using a computer with processor Intel Core i7 3.40 GHz and 12.0 GB RAM memory.

**4.1. Area Minimization.** Set trimming was employed to solve design examples aiming at the minimization of the heat transfer surface using the Kern and Bell-Delaware models. The optimal heat transfer area and the comparison of computational times between the proposed approach and mathematical programming (MILP<sup>13</sup> and ILP<sup>14</sup>) are shown in Table 2. The heat transfer area of all approaches are equal, because they always attain the global optimum. Comparisons presented by Gonçalves et al. between MILP<sup>13</sup> and previous MINLP approaches showed that the nonlinear formulations usually got trapped in local optima. The complete description of the solutions found using set trimming is displayed in the Supporting Information.

All approaches obtained solutions with the same heat transfer area because they are always able to identify the global optimum. However, as we can see in Table 2, there is a considerable difference in their computational performance. The set trimming technique was able to solve the optimization problems in a small fraction of the elapsed time consumed by the mathematical programming approaches. The design problems with Kern model were solved through set trimming demanding no more than 3.1% of the time consumed by mathematical programming using an ILP formulation. Equivalent tests involving the Bell-Delaware model indicated

gains even higher with the elapsed time for set trimming corresponding to a range of 0.3% to 1.1% of the time needed for the MILP solution.

Table 3 gives details about the number of initial candidates and the number of potential (although not yet necessarily feasible) candidates after each trimming step related to the Bell-Delaware model. It is possible to observe that the number of candidates is gradually reduced along the procedure. The last column indicates the number of configurations that are left after all the set trimming is performed. A final sorting of these feasible candidates is performed to identify the one with smaller area.

**4.2. TAC Minimization.** The comparison between set trimming and mathematical programming was also conducted for the minimization of the TAC as depicted in Table 4 for the Bell-Delaware model.<sup>13</sup> These optimization runs were executed with and without the pressure drop bounds, where the optimal

Table 4. Set Trimming versus Mathematical Programming: TAC Minimization (Bell-Delaware Model)

example	TAC (\$/y)	time (s) MILP	time (s) trimming	time (s) trimming without pressure drop
1	11735	433.25	1.63	1.50
2	7364	301.46	1.46	1.14
3	4129	186.69	1.17	1.09
4	19657	156.61	1.15	1.08
5	4912	409.25	1.10	1.09
6	6727	298.98	1.62	1.52
7	5663	159.39	1.28	1.25
8	15751	287.35	1.55	1.42
9	6866	357.13	1.66	1.54
10	7785	108.21	1.16	1.14

TAC values found were the same, with a slight difference in the computational times. The Supporting Information contains the details of each optimal solution found.

Similarly to the previous results for area minimization, the elapsed times to solve the problems using set trimming were much smaller than the ones employed to solve using mathematical programming through a MILP formulation (lower than 1.1%).

## 5. CONCLUSIONS

This paper presents the application of the set trimming technique for the design of shell and tube heat exchangers. This approach reduces the search space through the application of inequality constraints associated with problems with discrete variables. Typically, set trimming technique can be applied to design problems where each equipment candidate solution is defined by a set of discrete variables (e.g., commercial diameters, number of tubes, number of baffles, etc.). After the application of set trimming to all problem constraints, therefore identifying the feasible region of the problem, the optimal solution can be easily obtained by a sorting procedure. The search space of the design of shell and tube heat exchangers is essentially discrete, and it is interesting to observe that there are precedents in the literature of techniques that discretize continuous variables.<sup>26</sup>

The application of set trimming for the design of shell and tube heat exchangers indicates that the proposed approach can attain the global optimal solution in a small fraction of time (no more than 3.1%) of the corresponding solution using a mixed integer programming solver.

Future work will involve the analysis if the Set Trimming technique, proposed here to solve design problems with a given structure (e.g., pure discrete search space), can be extended to a more general approach. In relation to performance issues, future developments may consider how the features of each trimming and their interconnection can be investigated to provide a faster sequence of trimmings to reduce the computational effort.

## ■ ASSOCIATED CONTENT

### Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.iecr.0c02129>.

Complete description of the Kern and Bell-Delaware methods; description of the set of problems investigated; description of the optimal solutions found (PDF)

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## Notes

The authors declare no competing financial interest.

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## ■ NOMENCLATURE

$A$  = heat transfer area ( $\text{m}^2$ )  
 $\widehat{A}_{exc}$  = excess area (%)  
 $A_{req}$  = required area ( $\text{m}^2$ )  
 $B_c$  = baffle cut  
 $D_s$  = shell diameter (m)  
 $d_{te}$  = outer tube diameter (m)  
 $d_{ti}$  = inner tube diameter (m)  
 $F$  = correction factor for LMTD method  
 $h_s$  = shell-side convective heat transfer coefficient ( $\text{W}/\text{m}^2\text{K}$ )  
 $h_t$  = tube-side convective heat transfer coefficient ( $\text{W}/\text{m}^2\text{K}$ )  
 $k_{tube}$  = tube wall thermal conductivity ( $\text{W}/\text{m K}$ )  
 $L$  = tube length (m)  
 $lay$  = layout  
 $l_{bc}$  = baffle spacing (m)  
 $N_b$  = number of baffles  
 $N_{pt}$  = number of passes per tube  
 $N_{tt}$  = total number of tubes  
 $P$  = parameter used to calculate the correction factor  
 $\widehat{Q}$  = heat exchanger heat duty (W)  
 $R$  = parameter used to calculate the correction factor  
 $Re_s$  = shell-side Reynolds number  
 $Re_t$  = tube-side Reynolds number  
 $Re_{smax}$  = maximum shell-side Reynolds number  
 $Re_{smin}$  = minimum shell-side Reynolds number  
 $Re_{tmax}$  = maximum tube-side Reynolds number  
 $Re_{tmin}$  = minimum tube-side Reynolds number  
 $\widehat{R}_{fs}$  = shell-side fouling factor ( $\text{m}^2\text{K}/\text{W}$ )  
 $\widehat{R}_{ft}$  = tube-side fouling factor ( $\text{m}^2\text{K}/\text{W}$ )  
 $rp$  = tube pitch ratio  
 $TAC$  = total annualized cost ( $\$/\text{y}$ )  
 $\widehat{T}_{hi}$  = hot stream inlet temperature (K)  
 $\widehat{T}_{ho}$  = hot stream outlet temperature (K)  
 $\widehat{T}_{ci}$  = cold stream inlet temperature (K)  
 $\widehat{T}_{co}$  = cold stream outlet temperature (K)  
 $U$  = overall heat transfer coefficient ( $\text{W}/\text{m}^2\text{K}$ )  
 $v_s$  = shell-side flow velocity (m/s)  
 $v_t$  = tube-side flow velocity (m/s)  
 $v_{smax}$  = maximum shell-side flow velocity (m/s)  
 $v_{smin}$  = minimum shell-side flow velocity (m/s)  
 $v_{tmax}$  = maximum tube-side flow velocity (m/s)  
 $v_{tmin}$  = minimum tube-side flow velocity (m/s)  
 $\Delta P_s$  = shell-side pressure drop (Pa)  
 $\Delta P_t$  = tube-side pressure drop (Pa)  
 $\Delta P_{sdis}$  = shell-side available pressure drop (Pa)

$\Delta P_{tdisp}$  = tube-side available pressure drop (Pa)

$\Delta T_{lm}$  = logarithmic mean temperature (K)

## SETS

$S$  = initial set of candidates

$S_{LDmin}$  = set of candidates viable for minimum L/D ratio

$S_{LDmax}$  = set of candidates viable for maximum L/D ratio

$S_{LNbmin}$  = set of candidates viable for minimum baffle spacing constraint

$S_{LNbmax}$  = set of candidates viable for maximum baffle spacing constraint

$S_{vtmin}$  = set of candidates viable for minimum tube-side velocity constraint

$S_{vtmax}$  = set of candidates viable for maximum tube-side velocity constraint

$S_{vsmin}$  = set of candidates viable for minimum shell-side velocity constraint

$S_{vsmax}$  = set of candidates viable for maximum shell-side velocity constraint

$S_{Retmin}$  = set of candidates viable for minimum tube-side Reynolds number constraint

$S_{Retmax}$  = set of candidates viable for maximum tube-side Reynolds number constraint

$S_{Resmin}$  = set of candidates viable for minimum shell-side Reynolds number constraint

$S_{Resmax}$  = set of candidates viable for maximum shell-side Reynolds number constraint

$S_{DPtmax}$  = set of candidates viable for maximum tube-side pressure drop constraint

$S_{DPsmax}$  = set of candidates viable for maximum shell-side pressure drop constraint

$S_{Amin}$  = set of candidates viable for minimum area constraint

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